# The review of the habilitation of dr. Gyenis

**Summary**

Since his award of the PhD degree dr. Gyenis created an impressive body of work, which represents a significant contribution to formal epistemology. He combines in a creative and non-orthodox ways notions from different parts of formal epistemology and achieves in this way results which are novel, interesting and in many cases also surprising. The impact of Dr. Gyenis' work goes beyond pure formal epistemology, it is also relevant to the area of statistical learning, philosophy of science, mathematics and to theoretical computer science in general.

Dr. Gyenis published since his PhD remarkable number of more than 20 articles in the top internationally recognized journals devoted to the area of non-classical logics and formal epistemology (*Synthese*, *Journal of Philosophical Logic*, *Reviews of Symbolic Logic*, *Algebra Universalis*, *Logic Journal of IGPL*, *Math. Log. Quart*.). Moreover he co-authored two monographs and wrote several book chapters, some of them published in the prestigious *Springer* publishing house. He acquired an impressive amount of 125 citations and h-index 8 (Google Scholar). Dr. Gyenis gave talks at many international conferences and workshops, at a significant amount of them of them as an invited speaker. He is also well connected in the international scientific community as proven by his participation in several international projects.

In my opinion the achievements of the candidate clearly correspond to the requirements set out in the Law on Higher Education and Science.

***General framework and methodology***

The candidate included in his application a series of five thematically linked articles. These articles were published in journals that are highly recognized internationally (*Synthese*, *Journal of Philosophical Logic*, *Reviews of Symbolic Logic*, *Algebra Universalis*). The main topic of the habilitation is the Bayesian learning theory. Learning theory in general is a prominent topic in current formal epistemology, logic and computer science. Classical problem of learning in formal epistemology consists in exploring methods how a rational agent should learn or more precisely update her beliefs in the light of new information or evidence. In the case of full (i.e. certain) beliefs the rationality conditions are traditionally based on the laws of the classical logic and there are many frameworks providing different ways of representing updates (AGM style belief revision, Dynamic Epistemic Logic and others). When dealing with partial beliefs the rationality requirements are based on the rules of classical probability theory and the prominent learning method here is some form of conditionalization. This is also the basic method used in dr. Gyenis work.

While the most of recent approaches to modelling of partial beliefs take some existing system of epistemic logic as a background and add a probability on the top of it, Dr. Gyenis choose the opposite direction: he takes a standard system of probabilistic learning – namely Bayesian inference, introduces a modal logic based on this notion and studies its logical properties.

He introduces his modal system using possible world semantics usually called Kripke semantics named after Saul Kripke to whom the authorship is attributed (but it is worth noting that the very same idea appeared in the work of Hintikka and Kanger about the same time). Kripke semantics soon became a mainstream semantics not only for modal logics, but also for epistemic logics (logics representing belief and knowledge of rational agents). It consists of a set of elementary entities called possible worlds connected by a binary relation called accessibility relation. Necessity is then interpreted as a certain kind of uniformity: a statement is necessary in a particular possible world if it is true in all worlds accessible from this world. In the epistemic interpretation the relation between worlds is interpreted as epistemic possibility. A statement is known (believed) if it is true in all epistemically accessible worlds.

Dr. Gyenis interprets this relation in a novel way – he definins accessibility between possible worlds ("states" here ...) as a "learnability" based on the notion of bayesian inference. In particular a state is accessible from another one iff *it is possible* for an agent to infer/learn information contained in the second state from using a (one step) bayesian conditioning determined by her background probability. This interpretation of accessibility relation as learnability is close to the one used in dynamic logics, where the modal relation is understood as a transition from one state to another after performing some action (e.g. a computation changes the memory of a computer).

I would like to stress the following aspects of the dr. Gyenis approach which are novel from the methodological point of view:

- the idea to start with a purely probabilistic model of learning formalize it as a modal logic in contrast to mainstream approaches which start with an existing epistemic logic and introduce a probability measure on epistemic formulas
- originality of the framework the candidate introduces, in particular the interpretation of the accessibility relation known from normal modal logics as a possibility of transition from one state to another via Bayesian update

- a broad range of applicability of candidate's results – most of them have both technical and philosophical significance and are important not only to formal epistemology, which is the main research field of the candidate, but also to philosophy of science on one hand and mathematical logic on the other one

In the reminder of the review I comment on particular papers included in the habilitation in a more detail with a special attention to novelty and significance of the results, as well as their impact on the research area.

***General-properties-of-bayesian-learning-as-statistical-inference-determined-by-conditional-expectations(*** **(H5)**

The paper presents a background of the whole project. It investigates the properties of what the authors call "general Bayes learning" and shows its limits. This notion represents a method of inference using the method of conditional expectations which is a generalization of Bayes conditionalization – a standard method of probabilistic learning. This method of inference is used in the definition of Bayes accessibility relation on states, which can be seen as systems of partial beliefs of an agent. Accessibility of one state from another is defined as inferability by the method of conditional expectation.

Exploring the properties of Bayes accessibility relation gives surprising results: first not every two states are accessible from each other, moreover Bayesian learning is not reversible: if a state can be learned from another that represents evidence, then the converse is not true. The authors show even stronger result – the lack of weak connectedness. It informally means, that there exist states which are in a sense isolated – that an agent cannot learn via Bayesian statistical inference from no initial state. The definition of Bayes accessibility relation leads to another seemingly simple question: which probability distributions can an agent obtain from a given prior distribution *p*? The answer is surprising – not only there exist probability distributions which are not accessible in this way, but there are uncountably many of them.

The paper contains many more mainly technical results which are interesting and novel, which I will not discuss in this review. I would rather stress a serious consequence or both formal; epistemology and philosophy of science in general which can be drawn from these results: probabilistic learning even in its simple form is a seriously limited method in the sense made precise in the paper.

***The Modal Logic of Bayesian Belief Revision*** **(H3)**

The paper gives a nice introduction and motivation to the modal part of the whole project. It explains in a detail the ideas behind basic ingredients of the modal framework based on the notion of Bayes frame: states in the frame represent sets of partial beliefs i.e. various probability distributions over a fixed set of propositions. Modal accessibility relation between two states is (as before) defined as a possibility of inferring the probability distribution represented by the second state from the first state using conditioning on some proposition (evidence).

A terminological note: the authors use the notion “belief revision" in a broad sense to denote any change of beliefs of an agent, while in a large part of the literature it is almost exclusively used for one particular formalism, namely the infamous framework proposed by Alchurron, Gardenfors and Makinson and its various versions. However they clarify this issue in the introduction, so no confusion can arise.

The authors define different classes of Bayesian frames depending on their cardinality and explore the properties of the corresponding logics. They present several interesting results. They show that the logic of all Bayesian frames satisfy the axioms of one of the most well known logic S4, and the logics of finite and at most countable frames satisfy on the top of that the property of existence of endpoints (M axiom) and the property of non-existence of infinitely ascending chains (Grzegorczyk axiom). One of the important results of the paper is the theorem about the hierarchy of Bayes logics. The authors show that the class of all Bayesian frames is included in the class of infinite frames which is included in the class of finite frames. They also prove that these inclusions are proper. I find particularly interesting the correspondence between Bayes logic and Medvedev logic (called also the logic of finite problems). This fact is interesting not only from the point of view of formal epistemology, but also from a purely theoretical point of view of the theory of modal logics. The correspondence is used to prove the main results of the paper, namely that the modal logics of Bayes frame are not finitely axiomatizable. This result has not only a technical but also a philosophical significance. It shows that although the Bayesian learning is one of the most basic methods of learning in probabilistic frameworks, its logical representation is not straightforward and it is not easy to capture it by an axiomatic method.

***On the Modal Logic of Jeffrey Conditionalization*** **(H4)**

In this paper the candidate builds on the results of his previous paper (H3) in which he and his co-authors studied properties of a modal systems representing learning based on Bayes conditioning. The follow-up paper explore a similar modal framework in which the learning method is the one of

Jeffrey conditonalization. This method is more general in the sense that in the Bayesian case the agent learns from evidence which is certain (i.e. its probability changes to 1), while in the case of Jeffrey it is not – it only comes with a certain degree of probability. The candidate shows that the modal logics based on Jeffrey conditioning form a hierarchy similar to those based on Bayes conditioning. He discusses the question about the mutual relation between Jeffrey and Bayes logics. While in the case of finite frames the result is as expected: Jeffrey logics are properly contained in the Bayes logics, in the case of infinite frames the relation is much more tricky. The author conjectures that the classes of countable frames of both kinds coincide, but leaves this question as an open problem.

The central results of the paper concern axiomatizability of Jeffrey logics. The author shows, that similarly as in the case of Bayesian frames Jeffrey logics of both finite and countably infinite frames are not finitely axiomatizable. He aslo discusses the question of recursive axiomatizability and shows that it is closely connected to a longstanding open problem about recursive axiomatizability of Medvedev's logic. The results of this paper are novel and interesting and have again an impact on several areas outside formal epistemology.

***Standard-bayes-logic-is-not-finitely-axiomatizable*** **(H2)**

The candidate discussed in his previous article (H3) the problem of axiomatizability of Bayes logics he introduced. While this question was answered negatively for the case of finite Bayesian frames (i.e. those based on probabilities defined on a finite set of elementary propositions) the question of infinite frames remained open. This article provides an answer to this question. The author uses his previous result about coincidence of Medvedev and Bayesian frames of the same cardinality and some results and techniques from general theory of modal logics and shows, that the answer to the question of finite axiomatizability is negative also in the case of infinite Bayes frames. He also poses some open questions, the most interesting in my view is the problem if the logic of all Bayesian frames coincides with one of the most standard modal logics S4.

The results of this article are again significant not only for formal epistemology, but also for theoretical study of modal logics in general.

***Having a look at the Bayes Blind Spot*** **(H1)**

The article is devoted to investigation of properties of Bayesian Blind Spot, which was discussed also in the candidate's paper (H5). It denotes the set of probability of a Bayesian agent (set of degrees of belief) which the agent cannot learn in one step conditionalization, no matter what evidence she receives.  The authors show, that if the Boolean algebra of propositions which form the background of the agent's partial beliefs is finite, the Bayes Blind Spot has the cardinality of continuum. Moreover they demonstrate, that it is “big” not only in set-theoretical, but also in topological and measure theoretical sense. The properties of the Bayes Blind Spot are also studied in the context of several Bayesian models of learning. The authors discuss the problem of Bayes Blind Spot in the case of infinite Boolean algebra, but most of the questions remain an open problem in this case.

In the end of the article the authors stress, that the results of the article do not depend on the subjective interpretation of probability and therefore have significance not only for learning of Bayesian agents, but also for objectively understood statistical inference.

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