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Review of Zalán Gyenis' habilitation materials

The overarching theme of the work is a novel and very general perspective on Bayesian learning, which connects it with *prima facie* a quite different belief representation method, that of non-classical logic.

The main technical device is the theory of conditional expectations, of which Bayes' rule and Jeffrey's conditionalization are special cases. A major brilliant idea underlying what happens in the papers is that the relation of "being reachable by Bayesian learning" between states (linear functionals that encode probability measures by assigning expectation values to random variables) can be thought of as an accessibility relation, familiar from the possible world semantics of non-classical logics. This novel perspective opens up a whole array of research questions—most of the applicant's submitted work results from the identification and addressing of key questions in this area.

I found Dr Gyenis' work of excellent quality, both in terms of technical prowess and intellectual ingenuity. **I fully confirm that it deserves recognition in the form of habilitation.** What follows are brief comments on key findings present in the submitted works and my evaluation.

Paper 1., *General properties of Bayesian learning as statistical inference determined by conditional expectations*, introduces the approach and provides a range of results:

1. Bayes accessibility relation is reflexive, anti-symmetric, and non-transitive.
2. If every state is Bayes accessible from some other defined on the same set of random variables, then the set of states is called weakly Bayes connected.
3. The set of states is not weakly Bayes connected if the probability space is standard. The set of states is called weakly Bayes connectable if, given any state, the probability space can be extended in such a way that the given state becomes Bayes accessible from some other state in the extended space. Probability spaces are weakly Bayes connectable.

The key finding, that state spaces of standard probability spaces are not weakly connected, is very interesting: it points to the existence of Bayes inaccessible states ("blind spots") which can't be learned whatever the evidence, as long as the underlying space remains the same.

Paper 2., *The modal logic of Bayesian belief revision*, pushes further the connection between modal logic and probabilistic formal epistemology. The key move is to think of a measure q as possibly inferrable from p just in case there is some evidence E such that $q(\cdot) = p(\cdot|E)$ and of the set of possible worlds as the set of all probability measures on a measurable space. The authors define a hierarchy of modal logics (determined by the size restrictions on the underlying frames w.r.t. which validity is defined, and logical entailment between the size restrictions) that capture the formal properties of Bayesian belief revision. The resulting logics are interesting systems in the vicinity of **S4**, **S4.1** and **S4.Grz**. The authors, however, show that the resulting model logic determined by probabilities on a finite set of elementary propositions is not finitely axiomatizable.

Paper 3., *On the Modal Logic of Jeffrey Conditionalization* focuses on the more general Jeffrey formula and studies the corresponding modal logics. They turn out to be similar to the ones constructed from Bayes updating, up to indistinguishability if the underlying set of formulae is countably infinite. Jeffrey accessibility is reflexive and transitive (and so is an **S4** frame). Whether endpoints exist (and so, whether the frames are **S4.1**) depends on cardinality considerations.

Moreover, the findings of **Paper 2** generalize: the countable modal logics of Jeffrey belief revision determined by probabilities on a finite or countably infinite set of elementary propositions are not finitely axiomatizable.

Paper 4., *Standard Bayes logic is not finitely axiomatizable* extends the result in another direction. While the previous non-axiomatizability claims were proven to hold for probabilities on a finite set of elementary propositions, now the author proves that the modal logic of Bayesian belief revision determined by standard Borel spaces is also not finitely axiomatizable. This is a valuable claim, as it decouples non-axiomatizability considerations from somewhat arbitrary cardinality constraints.

Paper 5., *Having a look at the Bayes Blind Spot* is devoted to studying the measures that cannot be learned by a single conditionalization no matter what evidence an agent has. The authors prove that if the algebra is finite, the cardinality of the Bayes Blind Spot is a continuum, the same as the set of all probability measures. Moreover, the transitive closure of Bayesian updating doesn't provide much relief if the agent is conservative (i.e. keeps their original prior). If, on the other hand, they're bold (they take their posterior to be their new prior at each step), their Bayesian Blind Spot is going to be empty as soon as we allow for learning paths of length 2.

Evaluation and comments

The authors of this approach to belief revision take a unique perspective compared to the extensive literature on belief revision, such as AGM belief revision, dynamic epistemic logics, and dynamic logics. Rather than formulating plausible axioms for belief revision and studying their properties or comparing resulting systems, they analyze a real-life tool used in multiple applications of probabilistic learning through a logical lens.

The main new insight is the non-axiomatizability of mainstream probabilistic belief update methods. This means that no matter what axioms other approaches to belief revision propose, they will not be a complete set of axioms.

Another reason why studying the blind spots is interesting is that these results provide a commentary on another debate, that of appropriate methods for choosing priors for Bayesian inference. That is, the authors show that whatever method one employs for choosing priors, if the propositional knowledge of the agent is represented by a finite Boolean algebra, the Bayes Blind Spot is still going to be very large.

The author wishes to draw stronger critical conclusions for objective Bayesianism: if some objective state of affairs is in the Bayes Blind Spots, this objective state of affairs is supposed to be not inferrable from incomplete evidence.

One small concern is that it may well be that the Blind Spots for probability measures that take rational numbers as values are very small. The author points out that to claim that disregarding the measures that fail to satisfy this condition is epistemically irrelevant one would need to have a principled account of epistemic relevance.

While the two preceding arguments might have philosophical strength, a more practically oriented question remains. What is the extent to which a practicing Bayesian should worry about Blind Spots? An important question here would be: is it not the case that for any measure in a blind spot, there is a learnable measure not in the blind spot that is for all practical reasons close enough to it? This, however, is a somewhat extra-philosophical question which, while of interest for drawing practical lessons from the applicant's results, does not undermine the value of his achievement.

Sincerely,

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