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Review of the Habilitation Application of Dr. Bogdan Batko

Introduction. Dr. Bogdan Batko was awarded a Ph.D in Mathematics from the Pedagogical University of Kraków in 2000. His dissertation was entitled "Stability of alternative functional equations" and was supervised by Prof. Józef Tabor. From 2017 he has been working as an Assistant Professor at the Faculty of Mathematics and Computer Science of the Jagiellonian University. During 2020 – 2021 he held the position of Associate Research Professor at Rutgers University (USA). He has published 25 papers in the areas of (applied) dynamical systems and functional (equations) analysis. His first article appeared in 1999. In the years 2003-2009 and 2015-2023 he was a team member (Researcher) in various grants headed by Prof. Marian Mrozek, awarded by the Polish Scientific Research Committee (KBN), the Polish Ministry of Science and Higher Education, the 7th Framework Program of the EU, National Science Centre (Poland). In addition he was a team member in a grant of the Defense Advanced Research Projects Agency (USA) headed by Prof. Konstantin Mischaikow (2020–2021).

Assessment of the Applicant's scientific achievements. The Habilitation Application entitled "Conley index theory for discrete multi-valued dynamical systems" consists of 5 articles:

H1. B. Batko, M. Mrozek. Weak index pairs and the Conley index for discrete multi-valued dynamical systems, *SIAM J. Appl. Dyn. Syst.* **15** (2016), 1143-1162. DOI: 10.1137/15M1046691

H2. B. Batko. Weak index pairs and the Conley index for discrete multi-valued dynamical systems. Part II: properties of the Index, *SIAM J. Appl. Dyn. Syst.* **16** (2017), 1587-1617. DOI: 10.1137/16M1097584

H3. B. Batko, T. Kaczynski, M. Mrozek and Th. Wanner. Linking combinatorial and classical dynamics: Conley index and Morse decompositions. *Found. Comput. Math.* **20** (2020), 967-1012. DOI: 10.1007/s10208-020-09444-1.

H4. B. Batko, K. Mischaikow, M. Mrozek and M. Przybylski. Conley index approach to sampled dynamics. *SIAM J. Appl. Dyn. Syst.* **19** (2020), 665-704. DOI: 10.1137/19M1254404

H5. B. Batko. The Morse equation in the Conley index theory for discrete multi-valued dynamical systems, *J. Dyn. Diff. Equat.* (2022). DOI: 10.1007s10884-022-10136-3

Conley index theory, originating in the work of Charles Conley in the 70's ([Con78]), is geared towards the the analysis of the topological structure of invariant sets of smooth flows. When trying to apply this theory and practice, i.e. to samples originating from dynamical systems, one runs into problems of time and memory constraints. A natural approach is to bin the data. The advantage is that one can choose the bins so that the computations are feasible. However, this data reduction technique results with a multi-valued dynamical system, i.e. a map which sends each $x \in X$, where X is the phase space, to a subset $F(x) \in 2^X$, where 2^X is the power set of X.

The article [KM95] developed a Conley index theory for discrete multi-valued dynamical systems. However it included a definition of (strong) isolating neighborhoods (the distance between the invariant subset and the boundary is bounded below by the maximum over the diameters of $F(x)$, $x \in X$) which according to the Habilitation Summary was difficult to use in practice. The articles [H1] and [H2] set as an aim to remove this restriction. In [H1] the main innovations are the concepts of *isolating neighborhoods* and *weak index pairs* weakening the previously defined notions of *strong isolating neighborhoods* and *index pairs*. The paper manages to define a *Conley index* in this generality and includes an enlightening example. Important assumptions on F for the theory to work are that F is upper semi-continuous as well as F being *acyclic*, that is, it maps all $x \in X$ to spaces whose singular homology groups are the same as those of a point, e.g. contractible spaces such as Euclidean cubes (the acyclicity condition can be weakened).

The article [H2] establishes for the Conley index introduced in [H1] the *Ważewski property*, the *additivity property*, the *homotopy (continuation) property* as well as the *commutativity property* all known from classical Conley index theory. We will not repeat the exact definitions as this is well explained in the Habilitation Summary.

In [For98b, For98a] Forman developed a theory of combinatorial dynamical systems where he sought to transfer the theories of Morse [Mor34] and Conley [Con78] to the finite setting of a simplicial complex. Subsequently, this theory found many applications. It is natural to compare this well-established approach with the approach developed in the Habilitation. In [H3] the Applicant and his co-authors managed to achieve the best possible - a canonical correspondence between the combinatorial dynamics on a simplicial simplex and a well-defined multi-valued dynamical system on the geometric realization of this simplicial simplex (introduced previously in the paper [KMW16]). In particular, the correspondence is such that the *isolated invariant sets*, *Conley indices*, *Morse decompositions* and *Conley–Morse graphs* of the combinatorial dynamics (given by a *combinatorial vector field*) give rise to isomorphic objects for the multi-valued dynamical system on the geometric realization of this simplicial simplex. I find this result fundamental and very satisfying.

Interesting applications of the theory developed in the papers [H1] and [H2] is given in the paper [H4]. As an example let us consider [H4, Theorem 1.2]. The authors consider the Hénon map parameterized by a and b at the values $a = 1.65$ and $b = 0.1$. It should be remarked these are not the „classical parameters" $a = 1.4$ and $b = 0.3$ considered by Hénon. With the help of an explicit factor map on a subshift of finite type the theorem gives an explicit positive lower bound of the topological entropy of the attractor.

As mentioned the proof uses the theory of [H1] and [H2] but relies also on results on algorithmic aspects of Conley index theory from [MMP05], [Szy97] as well as from the PhD thesis of Mateusz Przybylski (2021) where the Applicant served as an auxiliary supervisor. Additionally the proof depends on certain computer-assisted computations which are not documented. Probably the authors assumed these to be easy to repeat but

in general, I believe that in such cases one should refer the reader to an online repository of the code and the computations.

The authors of [H4] do not unfortunately compare their results to known results in the literature. Such information is of course of utmost importance in this context. Are the results perhaps new because „untypical" values of a and b were considered? If so, how sensitive are the results to this choice? Is it possible to prove a similar result for a larger set of pairs simultaneously?

Classical Morse theory concerns the relation between Betti numbers of a manifold and *Morse indices* associated with a *Morse function* on the manifold ([Bot82]). In [CZ84] Conley and Zehnder developed an analogue approach in the context of Conley index theory of flows. The discrete-time dynamics case was achieved by Franks ([Fra82]) and Mrozek ([Mro91]). The paper [H5] further generalizes the theory, in particular the Morse equation and Morse inequalities to the context of multi-valued dynamical systems. Specifically, it is shown that if $\{M_1, M_2, \ldots, M_n\}$ is a Morse decomposition of an isolated invariant set S with respect to an upper semicontinuous $F : X \to 2^X$. Then

$$\sum_{i=1}^{n} p(t, M_i) = p(t, S) + (1 + t)Q(t),$$

where $p(t, M_i)$, $p(t, S)$ are the corresponding Poincaré series and Q is a formal power series with nonnegative integer coefficients. Addditional information $Q(t)$ is also given.

The proof is based on the method of Mrozek ([Mro91]). A simple example of obtaining dynamic information the Morse equation is given. However, it would have been more satisfying if a sophisticated application to a well-known example had been given.

One article out of the five articles of the Habilitation Application appeared in the flagship journal of computational mathematics, *Foundations of Computational Mathematics* - awarded the maximal number of points in the list of scientific journals of the Ministry of Science and Higher Education of Poland. Three articles appeared in the respectful journal *SIAM Journal on Applied Dynamical Systems*. Finally, one article of the five articles of the Habilitation Application appeared in the good journal *Journal of Dynamics and Differential Equations*. I assess the results of these papers as interesting and solid.

The remaining scientific achievements of the Applicant consist of 20 articles, published in mostly lower-quality journals. The contributions are within the theory of computational homology, the theory of Euler-Poincaré characteristic (applied within Conley index theory), as well as variations on the Hyers-Ulam stability theory.

In the last 25 years, the Applicant has published 25 articles and according to Web of Science, they have been cited 109 times (not including self-citations). One article per year is not a high number, but one should remember that the Applicant engaged in intensive didactic activity as we review below. The number of citations is not high, given the field in which the Applicant works, however, I consider it decent. It certainly shows that the work of the Applicant is read and has an impact.

In the last 26 years, the Applicant gave talks at 26 international conferences, most of them invited. This is a very satisfactory number. The Habilitation Application indicates rather a low level of international cooperation for the Applicant. However, one should note that the recent decade it has improved, where the pinnacle is a one-year stay at Rutgers University (USA) during 2020 – 2021 in the position of Associate Research Professor for the purpose of cooperating with one of the world most renown specialists in computational topology and Conley index theory, Prof. Konstantin Mischaikow.

The teaching and popularization of science as well as the organizational achievements of the Applicant are exemplary and should be praised in the highest terms. The activity of the Applicant in these areas permeates over four institutions with which the Applicant was associated, the Jagiellonian University, the Pedagogical University of Kraków, Wyższa Szkoła Biznesu – National-Louis University (WSB-NLU) and the State Higher Vocational School in Nowy Sącz. In addition, one should note that at the Pedagogical University of Kraków the Applicant served as the Vice Director for Student Affairs during 2014 – 2015, whereas at Wyższa Szkoła Biznesu – National-Louis University (WSB-NLU) the Applicant served as Vice Dean at the Department of Computer Science during 2004-2007.

At all of the above-mentioned institutions, the Applicant taught both as a lecturer and TA numerous courses such as *Mathematical analysis*, *Topology* and *Functional analysis* but also *MatLab programming*, *Mathematical modeling and computer simulations*, *Mathematics in economy* and *Cryptology*. The scope is very impressive. Another formidable achievement is the fact that the Applicant supervised over 100 master's and bachelor student theses in mathematics, computer science and management.

It has already been mentioned that the Applicant served as the Auxiliary Supervisor of the PhD thesis of Mateusz Przybylski (2021; PhD thesis: *Conley index theory methods in sampled dynamics*; Main Supervisor: Prof. Marian Mrozek).

It should be noted that the Applicant is the author of 3 textbooks: B. Batko, J. Malczak, *Mathematics in economy (in Polish)*, WSB-NLU, Nowy Sącz 2002; B. Batko, *Probability theory and statistics (in Polish)*, WSB-NLU, Nowy Sącz 2005; B. Batko, M. Mrozek, *Operations management (in Polish)*, WSB-NLU, Nowy Sącz 2006. The Applicant is also the author of several undergraduate study programs in mathematics and computer science. He also served as a coordinator of two projects related to undergraduate programs.

All of the above shows an intense engagement and notable commitment to the education of students.

Summary. The results presented in the Habilitation Articles constitute a significant contribution to the development of the dynamical systems discipline of Conley index theory both from abstract and applied perspectives. I am convinced that the scientific and research achievements presented by the Habilitation Applicant as well his teaching and international cooperation activities meet the statutory and customary requirements for granting a Habilitation. Therefore, I support awarding Dr. Bogdan Batko the degree of Habilitated Doctor in the mathematical sciences in the discipline of mathematics.

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