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Report on the habilitation thesis of Guillaume Olive

Candidate. Guillaume Olive has obtained a PhD degree in mathematics from Université Aix-Marseille in November 2013. He is a coauthor of 14 papers, according to the MathSciNet they are cited 186 times. The number of unique citing authors is 152. Finally, his results have been cited in 131 publications. His main area is control of partial differential equations. Recently he extended his interests also to the complex Monge-Ampére equations.

He had post-docs in Sorbonne, in San Diego, California, Bordeaux, and recently at UJ in Kraków. He received the SONATA grant from NCN.

He collaborated with distinguished mathematicians, among others: Jean-Michel Coron or Sławek Dinew.

Achievement. Guillaume Olive has decided to present the following set of articles as his habilitation achievement:

1. Boundary stabilization in finite time of one-dimensional linear hyperbolic balance laws with coefficients depending on time and space. This is a common paper with J.-M. Coron, L. Hu and P. Shang.

2. Minimal time for the exact controllability of one-dimensional first-order linear hyperbolic systems by one-sided boundary controls. This paper is written in collaboration with L. Hu.

3. Null controllability and finite-time stabilization in minimal time of one-dimensional first-order 2×2 linear hyperbolic systems. Common paper with L. Hu.

4. Equivalent one-dimensional first-order linear hyperbolic systems and range of the minimal null control time with respect to the internal coupling matrix. Again with L. Hu.

Papers 1. and 4. were published in J. Differential Equations which is a decent journal. Paper 3. appeared in a french journal ESAIM COCV, known in the field. Finally, paper 2. was published in the traditional J. Math. Pures Appl. founded in XIXth century by J. Liouville.

All the articles included in the achievement are related to the boundary control of linear hyperbolic equations in one spatial variable. More precisely, the main issue in the articles seems the estimate of the minimal time of control/stabilization. The presented papers are complemented by the description of the achievement and the declarations of the co-authors concerning the contribution of dr Guillaume Olive. Though the main area of my work are partial differential equations, I have also worked in optimal control for ordinary differential equations, I do not have any experience in control theory of partial differential equations. Hence a guided introduction to the topic and in particular, description of achievements, with particular emphasis on ideas, tricks and calculations that resulted in significantly new results, would have been very much appreciated. The description by G. Olive contains a brief introduction to the area, it seems however rather directed to the specialists in the field of control of PDEs. This is a pity that the introduction for the audience being familiar rather with PDEs than the peculiar area of control of PDEs is missing. In particular, it is not easy at all to judge the influence of the author on the general field of mathematics or other branches of science. The control of PDEs seems very applicable field, but I haven't learnt any detailed examples of the utility of the results of an author from his description. Though some of the exact formulas of the minimal control time given by the author are really explicit, he does not point any applications in concrete real world problems. I could find general comments that control of PDEs is applicable in '...many areas of science...', showing some concrete examples from the real world applications would have been really nice. Next, concerning the discussion of the advantages in the field of control of PDEs, my impression, after reading author's description, is that the main achievement is an extension of a theory from the problems concerning the equations with coefficients depending on spatial variables only to the ones depending on both, space and time, on the one hand, and on the other hand the extension of the results for conservation laws to the case of balance laws. It is said that such extensions are important, but I'm missing the examples of such importance. I noticed that some of the exact formulas obtained by the author are particularly straightforward, so in particular could be used in the peculiar problems. The author makes an effort to explain the novelty of his results. Usually it based on long reasonings, often involving detailed techniques in integral equations, like the Titschmarsch theorem or advanced linear algebra tricks. What I haven't learnt from the author's description is what sort of original computations, tricks, methods were introduced. I appreciate such contribution the most. My impression is also that such 'small' steps often influence the area. Finally, let me mention that despite all the coauthors seem to appreciate the collaboration with Guillaume Olive and they consider his contributions valuable, I am missing the reference to the specific calculations, lemmas or at least general ideas coming from G. Olive within the collaboration. Next, I would also like to learn more about the almost empty period in the career of G.Olive, years 2017, 2018.

Let me now briefly discuss the content of the main achievement.

All papers 1.-4. deal with the problem of identifying the minimal time at which the boundary control of a system of spatially one-dimensional system of linear balance laws with coefficients depending on both, space and time, steers it towards a desired state (if the target is 0, we call such control a null control). Control is restricted to the one side of the boundary. And only to the values on the boundary of the characteristics with a fixed velocity sign. As far as I understand, the situation concerning the conservation laws instead of balance laws is well understood regarding the minimal control time. The author also suggests that in the case of balance laws the results concerning minimal control time were satisfactory in the case of coefficients depending on space only.

In the first paper the main result gives the lower bound of the minimal time of control stability (see the Def. 1.1 i 1.) for the system with time and space dependent coefficients. It takes an exact value in the time-independent case, showing that the result is an extension of the time-independent one present in the literature. The proof is an extension of the previous result of similar authors concerning the time-independent case. Some technical issues need to be resolved. It requires a deep knowledge of linear balance laws, integral equations, specially the Volterra equation and Fredholm equations and a non-trivial linear algebra.

The second paper is devoted to the proof that the exact controllability time (this time a problem with coefficients depending only on space) is finite provided the rank of some matrix is of full row rank. Moreover, what seems the most interesting, it gives an exact formula for such time in the situation when it is finite. This exact formula seems to be easily calculable in some examples. The main steps of the proof are related to the advanced linear algebra and the LU decomposition of matrices. Some non-trivial preparatory steps involving canonical forms of non-quadratic matrices are involved. They seem to be related to the recent progress in linear algebra.

In the third article, (again space dependent coefficients) the authors study the restricted case (the assumptions on the considered hyperbolic system are stronger). The authors give the minimal time for null controls in the studied case. The obtained formula seems easily calculable in many cases. The proof required several technical improvements of classical methods, like the backstepping method.

Finally, in the paper 4. the smallest and largest values of the control minimal time that can be taken with respect to the internal coupling matrix are studied. The long and technical proof again relies on backstepping method.

Other contributions. Let me briefly discuss other results of G. Olive, not included in his achievement. First, some of his papers are devoted to the studies of contollability, stability as well as the studies of minimal time of control for different type of equations, like linear integro-differential hyperbolic or linear parabolic equations (the latter in his PhD thesis for instance). Quite often the candidate applies there some variants of backstepping method.

Next, there are two papers on controllability which deserve being mentioned separately. First, a 2017 paper with Alabau-Boussouira and Coron. It concerns a controllability of a quasilinear system of conservation laws. All the other controllability results of Olive concern the linear systems. Here, the system is nonlinear. An application of the celebrated Nash-Moser scheme is worth mentioning. The second paper of a separate nature is a more general result in a common paper with M. Duprez, 'Compact perturbations of controlled systems'. It gives a necessary condition for the linear system in a very general framework to be controllable in the finite time T. Apparently, the result can be applied in many cases of linear equations.

Finally, G.Olive essentially extended his interests within last years towards the complex Monge-Ampére equation. He already authored two papers in this area. Here, completely different approach, involving different techniques and theories is required.

Summary. G. Olive clearly satisfies all the formal numerical requirements for the habilitation candidate in the Polish system, involving the number of papers, the quality of journals, where papers were published or the number of citations gained by the results of the candidate. Still, one should also make sure that his achievement is a serious advance in the area. Despite the recent reality, where papers are often coauthored by many persons, it would not do bad if it was clear what exactly are the contributions of the candidate. It would also be nice if we could tell that the area covered by the work of the candidate is broad. On the one hand, both PhD thesis, as well as the main part of the further activity of G. Olive were devoted to the topic of boundary controllability of linear systems of PDEs. Still, he has been working in many types of systems of PDEs. Next, he authored a paper on controllability of nonlinear conservation laws, also a contribution on a general system of linear equations is present among his scientific achievements. The latter, together with a recent contribution in the completely separate area of complex Monge-Ampére equation seem to fulfil the requirement establishing a scientific maturity.

Since I am not a specialist in control of PDEs, it is an extremely difficult task then to judge the essentiality of the results obtained by the candidate. I would like to appeal for an official meeting (kolokwium habilitacyjne), during which a candidate could address the following three issues, which would clearly help me to conclude my report in an affirmative way:

1) What is the concrete motivation of an extension of the theory from spatial dependence of coefficients to the time-space dependence in 1.? Were there some problems in the literature requiring such extension? Mathematical or real world problems? Similar question concerning the extension from conservation laws to balance laws.

1) Is it possible to point some 'small' new calculations, tricks, lemmas which were found by G.Olive and his collaborators when extending the theory contained in his achievement? Can some of them be useful in other situations? Or are they peculiar to the studied case?

2) Is it possible to identify some calculations, lemmas, tricks or general ideas which came exactly from G.Olive, not necessarily during the scientific discussions with collaborators?

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