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Report on the habilitation thesis of Dr. Yizhuang Liu

Dear Colleagues,

This report evaluates the habilitation thesis entitled "Euclidean formulation of parton distribution functions: factorization, evolution and lattice applications" submitted by Dr. Yizhuang Liu in fulfillment of the requirements for the degree of habilitated doctor in the field of exact and natural sciences in the discipline of physical sciences at the Jagiellonian University.

The work builds on the author's collaborative research published in seven peer-reviewed papers (2020-2023) from leading journals in the field (Phys. Rev. D, Nucl. Phys. B, Phys. Rev. Lett. B, Rev. Mod. Phys.). Dr. Liu's contribution to these publications was crucial, as evidenced by the letters from his collaborators.

The thesis delves into the novel approach to computation of Transverse Momentum Dependent (TMD) Parton Distribution Functions (PDFs) within the framework of Quantum Chromodynamics (QCD). These functions offer a powerful tool for understanding the structure of hadrons, calculating high-energy scattering processes, and advancing our knowledge of the strong force that binds quarks and gluons together. TMDs are characterized by their dependence on both the longitudinal momentum fraction and the intrinsic transverse momentum. Unlike integrated PDFs, TMDs provide additional information about the parton's transverse motion within the hadron. They are essential for describing observables involving transverse momentum dependence.

TMDs are nonperturbative quantities and their computation in QCD remains an extremely challenging open problem. Our current understanding of TMD's in QCD relies on the concept of factorization. This powerful technique separates the contributions from different physical scales within the problem (hard, soft, and collinear) into distinct functions. We can then leverage perturbative QCD to predict how these functions depend on the relevant energy scales. However, TMD factorization is significantly more intricate compared to conventional collinear factorization due to the emergence of rapidity divergences. To handle these divergences, an additional energy scale (rapidity regulator) and an extra subtraction term need to be introduced into the factorization formula.

There has been a surge in research focused on computing parton distribution functions using lattice QCD methods. Defining them in lattice QCD presents significant challenges due to several theoretical and computational hurdles. Parton distribution functions are inherently linked to Minkowski (Lorentzian) spacetime, while lattice QCD calculations are performed in Euclidean spacetime. A recent proposal suggests that the PDFs can be computed by taking the large-momentum limit of the so-called quasi-PDFs which are defined as correlators in Euclidean space. This equivalence between Euclidean and physical regimes paves the way for a large-momentum effective theory. This thesis extends the previous analysis and proposes an Euclidean formulation of TMD distributions.

The thesis demonstrates that the physical TMD distribution can be recovered from a quasi-TMD within the Euclidean framework. The quasi-TMD is constructed from a hadronic matrix element bilocal quark-antiquark operator, where gauge links are inserted to ensure gauge invariance. Similar to the rapidity divergences of TMD distributions, quasi-TMDs also exhibit specific divergences. These divergences scale linearly with the length of the integration contour within the gauge links. They can be removed by dividing the quasi-TMD by a square-root of a rectangular space-like Wilson loop. By subtracting these divergences, the quasi-TMDs become well-defined in the limit as the integration contour length approaches infinity. This allows the author to analyze the resulting quantity using standard factorization techniques.

The factorization formula presented in equation (55) of the thesis establishes the relation between the physical TMDs and quasi-TMDs in the large energy limit. It involves two perturbative functions that describe the contribution of hard and collinear emissions, as well as a nonperturbative soft function (referred to as the "reduced soft factor") that takes into account the emission of soft gluons. The perturbative functions satisfy the evolution equations, which involve the universal cusp anomalous dimension and a set of process-dependent anomalous dimensions. By solving these equations, the author derived an improved factorization formula (75) that resums all large logarithmic corrections to the quasi-TMDs.

The factorization formulas, like equations (55) and (75), exhibit an interesting property: they are independent of the specific hadronic state being probed. This implies that the same fundamental formula applies to observables such as light-meson form factors and hadronic distribution amplitudes. In all these cases, the expressions for these quantities involve the same hard, collinear, and soft functions used for TMDs.

Recognizing this universality, the author leveraged it to derive an expression for the non-perturbative soft function (equation (103)) in terms of light-meson form factors and meson distribution amplitudes. This relationship is significant because it opens a path to calculating the soft function using lattice QCD. This calculation is possible by exploiting the well-established connection between Euclidean space observables (used in lattice QCD) and their counterparts in Minkowski space (relevant for physical measurements) in the high-energy limit.

Section 3 of the thesis describes how the theoretical framework can be used for lattice calculations of TMDs. Building upon the author's formulation of TMD distributions in Euclidean space, several research groups have successfully performed direct lattice simulations. These simulations have yielded promising preliminary results, demonstrating the potential of this approach.

The lattice formulation of TMDs presented in the thesis offers a powerful tool for overcoming the limitations of traditional methods. This leads to a more comprehensive understanding of hadron structure, improved accuracy in TMD calculations, and the ability to make first-principle predictions for important physical processes, such as Drell-Yan and Higgs production at low transverse momentum.

In summary, Dr. Liu's thesis demonstrates a deep understanding of QCD and presents significant original contributions through Euclidean formulation of parton distribution functions. His publications in the leading peer-reviewed journals demonstrate his expertise and ability to disseminate complex research findings. I therefore enthusiastically recommend granting to Dr. Yizhuang Liu the degree of habilitated doctor.

Sincerely yours,

G. Korchemsky

Gregory Korchemsky
Directeur de Recherche